AD NUMBER

AD850232

NEW LIMITATION CHANGE

TO

Approved for public release, distribution unlimited

FROM

Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; Mar 1969. Other requests shall be referred to Commanding Officer, Naval Weapons Center, Corona Lab., Attn: Code 2035, Corona, CA 91720.

AUTHORITY

US Naval Weapons Center ltr dtd 16 Apr 1969

NWCCL TP 845

AD850232

THE LINVILL METHOD OF HIGH FREQUENCY TRANSISTOR AMPLIFIER DESIGN

by

D. Rubin

Research Department

ABSTRACT. This report discusses a method—known as the Linvill method—for determining the terminations that a transistor amplifier should have for a specified value of power gain and bandwidth. Basically, the Linvill method makes use of measured transistor parameters to develop charts from which one can read power gain and input impedance or admittance as functions of the load terminations. This report gives a complete geometrical derivation of the Linville "stability factor," whose value is an indication of the stability of the amplifier under various load conditions. In addition, procedure steps are given for using the charts developed for determining input and load admittances.

NAVAL WEAPONS CENTER

CORONA LABORATORIES, CORONA, CALIFORNIA * MARCH 1969

91720

35

NAVAL WEAPONS CENTER
CORONA LABORATORIES
R. S. Forbis, CDR., USN .... Commanding Officer

FOREWORD

In designing high frequency receivers, personnel of the Electronics Division of the Naval Weapons Center Corona Laboratories must devise sensitive amplifiers which have very little noise and wide bandwidth. Such circuits require highly effective use of transistors. This report describes a method by which the measured admittance parameters of a transistor are used to obtain power gain as a function of frequency and load admittances of an amplifier.

This work was accomplished under AIRTASK A36533/216/69F-17343604.

Released by
C. P. HABER
Acting Associate Head of
Research Department, Corona
26 March 1969

Under authority of
H. W. HUNTER, Head
Naval Weapons Center
Research Department

NWC Corona Laboratories Technical Publication 845

Published by . . . . Technical Information Division
Collation . . . . Cover, 17 leaves, DD Form 1473, abstract cards
First printing . . . . 75 unnumbered copies
Security classification . . . . UNCLASSIFIED

CONTENTS

INTRODUCTION

It is not reasonable to use transistor models for the design of amplifiers in the UHF region and above. Actual measurement of the admittance (y), hybrid (h), or scattering (s) parameters of a transistor at the frequencies of interest is practical with laboratory instruments and leads to precise calculations of gain, bandwidth, input and output admittances, etc. In 1956 Linvill and Schimpf (Ref. 1) described a method by which gain and input impedance could be characterized graphically for any value of load admittance. All that was required was a knowledge of the h parameters at the frequencies of interest. From these, radii of various circles (gain circles) were calculated and positioned on a regular Smith Chart. The regions of stability were also clearly indicated. The method, which was reiterated in a book (Ref. 2), was reviewed by various writers (Ref. 3 through 6), most of whom preferred the use of y parameters. An exceedingly important quantity known as the stability factor C (called the critical factor by Linvill) was defined by Linvill in his 1956 paper. The C factor, a function only of the device parameters, determined if the device would be stable for all values of load admittance, or if not, would determine the area of the Linvill chart (and hence the range of load values) to be avoided.

Linvill and his reviewers define the C factor, but do not give explicit proof regarding its evaluation, although it was suggested that the C factor could be evaluated by way of geometry. In this report it will be shown that the geometry used in the discussion of the Linvill method could indeed be used to determine the value of the C factor. The explanation of the Linvill method can then be made without it being accepted on definition alone or requiring additional references. It will finally be shown how charts obtained by the Linvill method can be used to determine power gain and input admittance as a function of load admittance.

POWER RELATIONSHIPS

Equations relating power input (P_i) and power output (P_o) are obtained from the relationships between small voltage and current phasors (e,i) and the y or h parameters of the device. Scattering (s) parameters may be directly converted to either of these two sets. The following analysis will be done using y parameters. The derivation of the power equations using h parameters is given in Appendix A. By definition (see Fig. 1),

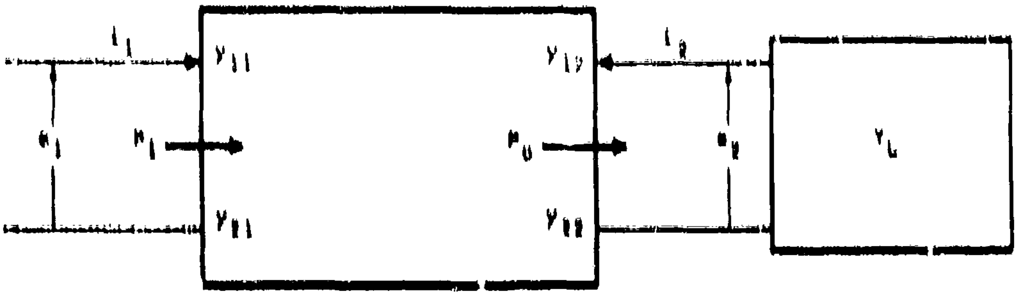

FIG. 1. Circuit Parameter Diagram.

$$i_1 = y_{11}e_1 + y_{12}e_2$$

$$i_2 = y_{21}e_1 + y_{22}e_2 = -e_2Y_L$$

where Y_L is the load admittance. Therefore,

$$\frac{e_2}{e_1} = -\frac{y_{21}}{y_{22} + Y_L}$$

If Y_L were set $= y_{22}^*$, then

$$\left(\frac{e_2}{e_1}\right)_{Y_L = y_{22}^*} = \frac{-y_{21}}{y_{22} + y_{22}^*} = \frac{-y_{21}}{2\text{Re } y_{22}} \tag{1}$$

Therefore, in general,

$$\frac{e_2}{e_1} = (L + jM)\,\frac{-y_{21}}{2\mathrm{Re}\,y_{22}} \tag{2}$$

where L and M are real. Since $P_i = \mathrm{Re}(i_1 e_1^*)$, where i_1 and e_1 are in rms values,

$$P_i = \mathrm{Re}\left[\left(y_{11}e_1 + y_{12}e_2\right)e_1^*\right] = \mathrm{Re}\left[y_{11}\left|e_1\right|^2 + y_{12}(L + jM)\left(\frac{-y_{21}}{2\mathrm{Re}\,y_{22}}\right)\left|e_1\right|^2\right]$$

Now, let

$$P_i' \equiv \frac{P_i}{\left|e_1\right|^2} = \mathrm{Re}\left\{y_{11} + (L + jM)\left[\mathrm{Re}\left(\frac{-y_{12}y_{21}}{2\mathrm{Re}\,y_{22}}\right) + j\mathrm{Im}\left(\frac{-y_{12}y_{21}}{2\mathrm{Re}\,y_{22}}\right)\right]\right\}$$

Then

$$P_i' = \mathrm{Re}\,y_{11} - L\,\mathrm{Re}\left(\frac{y_{12}y_{21}}{2\mathrm{Re}\,y_{22}}\right) + M\,\mathrm{Im}\left(\frac{y_{12}y_{21}}{2\mathrm{Re}\,y_{22}}\right)$$

also

$$P_o = -\mathrm{Re}\left(i_2^* e_2\right) = -\mathrm{Re}\left[\left(y_{21}^* e_1^* + y_{22}^* e_2^*\right)e_2\right]$$

$$= -\mathrm{Re}\left[y_{21}^* e_1^* (L + jM)\left(\frac{-y_{21}}{2\mathrm{Re}\,y_{22}}\right)e_1 + y_{22}^*\left|e_2\right|^2\right] \tag{3}$$

but

$$\left|e_2\right|^2 = e_2 e_2^* = (L + jM)\left(\frac{-y_{21}}{2\mathrm{Re}\, y_{22}}\right)(L - jM)\left(\frac{-y_{21}^*}{2\mathrm{Re}\, y_{22}}\right)\left|e_1\right|^2$$

$$= \frac{L^2 + M^2}{4\left(\mathrm{Re}\, y_{22}\right)^2}\left|y_{21}\right|^2\left|e_1\right|^2$$

We next define P_o' where

$$P_o' \equiv \frac{P_o}{\left|e_1\right|^2} = -\mathrm{Re}\left[\frac{-\left|y_{21}\right|^2}{2\mathrm{Re}\, y_{22}}(L + jM) + \frac{y_{22}^*\left(L^2 + M^2\right)\left|y_{21}\right|^2}{4\left(\mathrm{Re}\, y_{22}\right)^2}\right]$$

$$= -\mathrm{Re}\left[\frac{-\left|y_{21}\right|^2}{2\mathrm{Re}\, y_{22}}(L + jM) + \left(\mathrm{Re}\, y_{22} - j\mathrm{Im}\, y_{22}\right)\frac{\left(L^2 + M^2\right)\left|y_{21}\right|^2}{4\left(\mathrm{Re}\, y_{22}\right)^2}\right]$$

Therefore,

$$P_o' = \frac{L\left|y_{21}\right|^2}{2\mathrm{Re}\, y_{22}} - \frac{\left(L^2 + M^2\right)\left|y_{21}\right|^2}{4\mathrm{Re}\, y_{22}} \tag{4}$$

We finally let

$$a = \frac{\left(y_{12}y_{21}\right)}{2\mathrm{Re}\, y_{22}} \qquad a_1 = \frac{\mathrm{Re}\left(y_{12}y_{21}\right)}{2\mathrm{Re}\, y_{22}} \qquad a_2 = \frac{\mathrm{Im}\left(y_{12}y_{21}\right)}{2\mathrm{Re}\, y_{22}}$$

$$b_1 = \frac{\left|y_{21}\right|^2}{2\mathrm{Re}\, y_{22}} \tag{5}$$

then from Eq. 3 and 4,

$$P_i' = \mathrm{Re}\, y_{11} - La_1 + Ma_2 \tag{6}$$

$$P_o' = Lb_1 - \left(L^2 + M^2\right)\frac{b_1}{2} \tag{7}$$

The constants L and M are functions of Y_L, since from Eq. 1 and 2

$$\frac{e_2}{e_1} = \frac{-y_{21}}{y_{22} + Y_L} = (L + jM)\left(\frac{-y_{21}}{2\mathrm{Re}\, y_{22}}\right)$$

or

$$Y_L = \frac{2\mathrm{Re}\, y_{22}}{L + jM} - y_{22} \tag{8}$$

That is, a given value of L and M represents a specific Y_L. For $Y_L = y_{22}^*$, we have from Eq. 8

$$Y_L = y_{22}^* = \frac{2\mathrm{Re}\, y_{22}}{L + jM} - \left(\mathrm{Re}\, y_{22} + j\mathrm{Im}\, y_{22}\right)$$

$$y_{22}^* = \left(\mathrm{Re}\, y_{22} - j\mathrm{Im}\, y_{22}\right) = \mathrm{Re}\, y_{22}\left(\frac{2}{L + jM} - 1\right) - j\mathrm{Im}\, y_{22}$$

Therefore

$$\frac{2}{L + jM} - 1 = 1$$

or $L = 1$ and $M = 0$. Consequently, for $Y_L = y_{22}^*$,

$$P'_i = P'_{io} = \mathrm{Re}\, y_{11} - a_1$$

$$P'_o = p'_{oo} = \frac{b_1}{2} \tag{9}$$

and gain

$$G_{oo} = \frac{P'_{oo}}{P'_{io}}$$

P'_o is therefore, from Eq. 7 and 9,

$$P'_o = 2P'_{oo}L - \left(L^2 + M^2\right)P'_{oo}$$

$$= P'_{oo}\left\{1 - \left[(L - 1)^2 + M^2\right]\right\} \qquad (10)$$

The equation for P'_o is that of a paraboloid whose axis is along the line $L = 1$, $M = 0$, and whose maximum height is P'_{oo}. Its intersection with the LM-plane (see Fig. 2) occurs when $P'_o = 0$. There we have $(L^2 - 1)^2 + M^2 = 1$, i.e., a circle of radius 1 centered at ($L = 1$, $M = 0$).

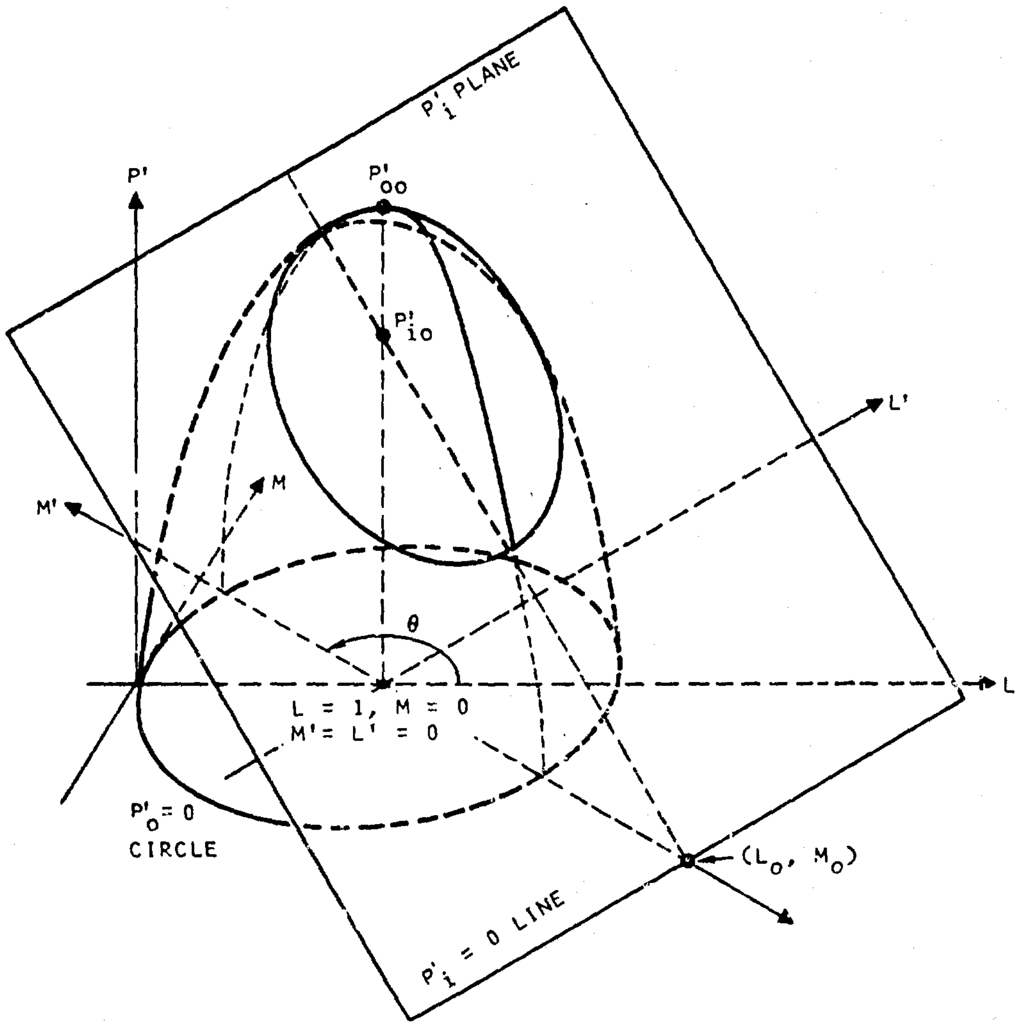

FIG. 2. Power Output and Input as Functions of L and M.

The equation for P'_1 is that of a plane, intersecting the LM-plane along the line given when $P'_1 = 0$, i.e., when $0 = \mathrm{Re}\, y_{11} - La_1 + Ma_2$, or

$$M = \frac{a_1}{a_2} - \frac{\mathrm{Re}\, y_{11}}{a_2} \tag{11}$$

There is another line, M' (called the gradient line), which lies in the LM-plane, is perpendicular to the $P_i' = 0$ line, and intersects ($L = 1$, $M = 0$). This line is the projection of a line in the P_i' plane along the maximum slope of the plane.

The gradient line may be written in the slope-intercept form $M = mL + d$. For this line to be perpendicular to the $P_i' = 0$ line, the slopes of the two lines must be negative recipricals of each other, i.e.,

$$m = -\frac{1}{\frac{a_1}{a_2}} = -\frac{a_2}{a_1}$$

so that

$$M = -\frac{a_2}{a_1} L + d$$

For $M = 0$ we must have $L = 1$, so that $d = a_2/a_1$. Hence the equation for the gradient line is

$$M = -\frac{a_2}{a_1}(L - 1) \tag{12}$$

The $P_i' = 0$ line and the gradient line intersect at (L_o, M_o). From Eq. 11 and 12,

$$M_o - \left(\frac{a_1}{a_2}\right) L_o = -\frac{\mathrm{Re}\, y_{11}}{a_2}$$

$$M_o + \left(\frac{a_2}{a_1}\right) L_o = \frac{a_2}{a_1}$$

Solving for M_o and L_o, we have

$$L_o = \frac{\left(\frac{a_2}{a_1}\right)^2 + \frac{\mathrm{Re}\, y_{11}}{a_1}}{1 + \left(\frac{a_2}{a_1}\right)^2}$$

$$M_o = \frac{a_2}{a_1}\left(1 - L_o\right) = \frac{a_2}{a_1} \frac{\left(1 - \frac{\mathrm{Re}\, y_{11}}{a_1}\right)}{\left(\frac{a_1}{a_2}\right)^2 + 1} \tag{13}$$

The $P_o' = 0$ circle, the $P_i' = 0$ line, and the gradient line are shown in Fig. 3. From the figure

$$\tan\theta = -\tan\phi = -\frac{M_1}{1} \tag{14}$$

and from Eq. 12, when $L = 0$, $M = a_2/a_1$; therefore

$$\tan\theta = -\frac{a_2}{a_1} \tag{15}$$

and since

$$\left(-y_{12}y_{21}\right)^* = \left[-\mathrm{Re}\left(y_{12}y_{21}\right) - j\mathrm{Im}\left(y_{12}y_{21}\right)\right]^*$$

$$= -\mathrm{Re}\left(y_{12}y_{21}\right) + j\mathrm{Im}\left(y_{12}y_{21}\right)$$

the equation for θ in terms of y_{12} and y_{21} is, from Eq. 5,

$$\theta = \tan^{-1} \frac{\mathrm{Im}\left(y_{12}y_{21}\right)}{-\mathrm{Re}\left(y_{12}y_{21}\right)}$$

or

$$\boxed{\theta = \tan^{-1} \mathrm{Arg}\left(-y_{12}y_{21}\right)^*} \tag{16}$$

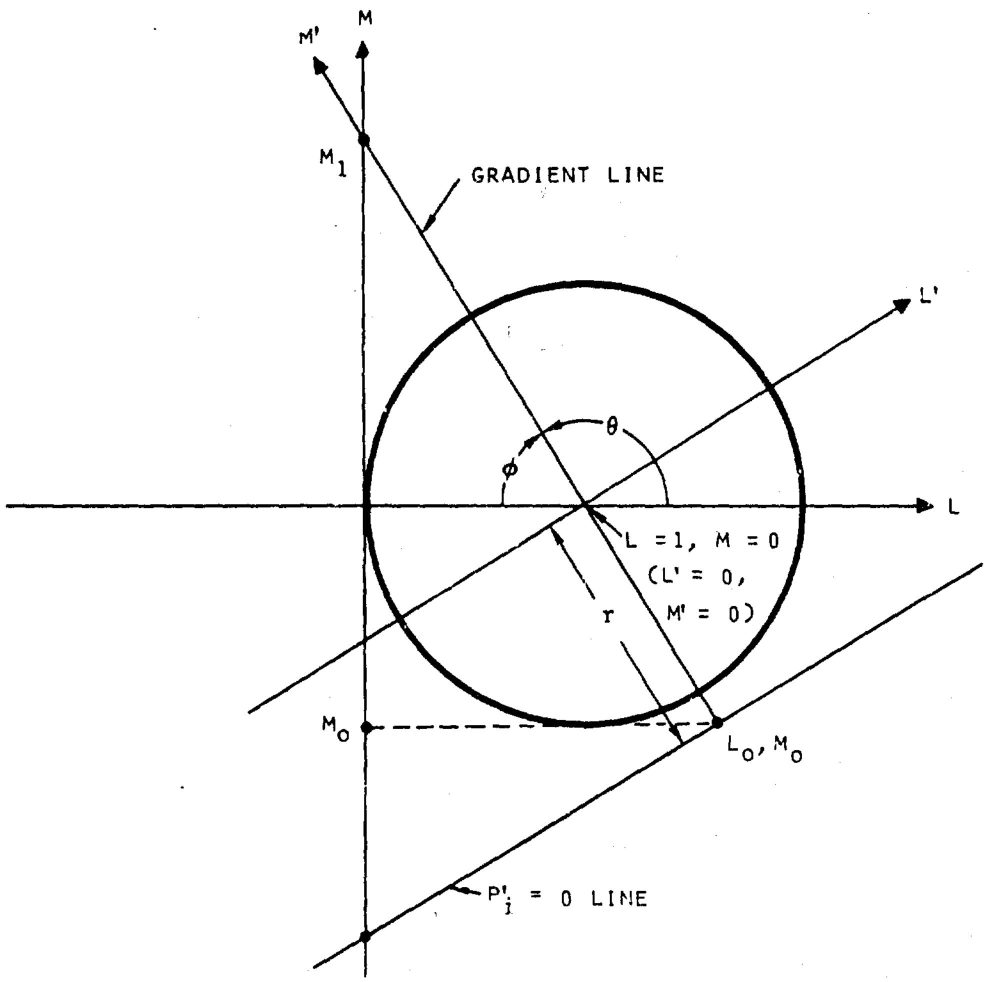

FIG. 3. LM-Plane Traces.

Any line perpendicular to the LM-plane which intersects the P_i' plane and P_o' paraboloid will give the gain $G = P_o/P_i = P_o'/P_i'$ for a particular value of L and M, and hence for a particular Y_L. For a given output power (a circle parallel to the LM-plane), the point of least power input must lie along the line of steepest descent of the P_i' plane. To find the maximum gain, we find the point along the M' axis which gives the largest P_o' over P_i' ratio.

STABILITY FACTOR

Sighting down the L' axis of Fig. 2, we obtain the projection shown in Fig. 4. The P_i' plane is reduced to a line and the P_o' paraboloid reduced to a parabola. Note from Fig. 2 that instability can occur for some values of L and M if the $P_i' = 0$ line intersects the $P_o' = 0$ circle. Within the sector thus formed, there will be some values of P_o' even though P_i' is zero or negative, i.e., the device will oscillate for some values of Y_L. In Fig. 4, if $-r < -1$ the device will be stable for all Y_L. From this figure it is clear that for potential instability $C \geq 1$; this is the C referred to earlier as the "stability factor." It will be shown that C depends only on the device parameters (not Y_L). In general there will be some frequency above which C will be less than one and (because of low inherent gain) the device cannot oscillate.

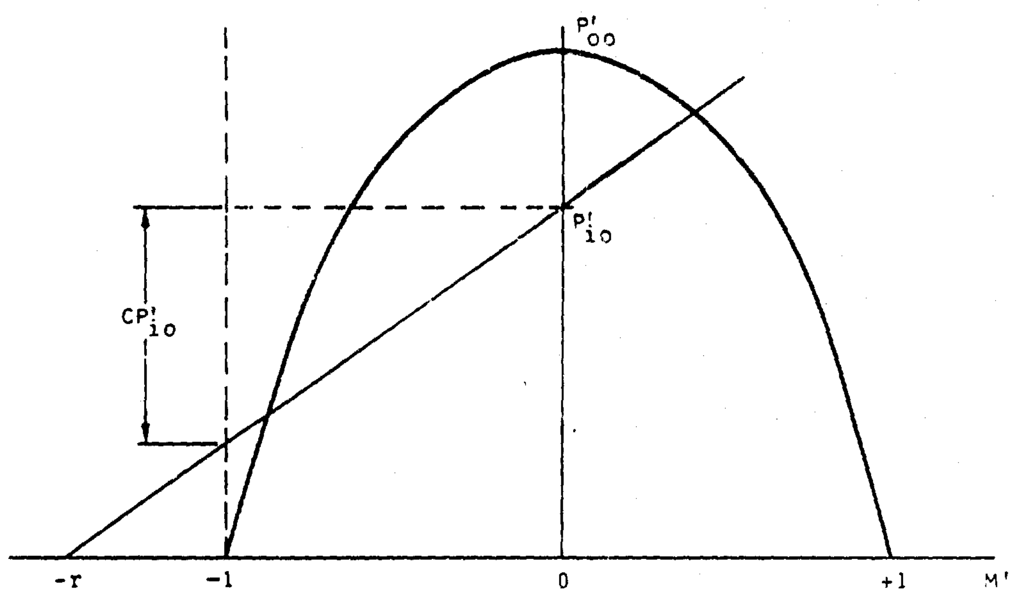

FIG. 4. Gain Diagram.

We next derive the equation for C. The distance r is found from Fig. 2 to be

$$r = \sqrt{(L_o - 1)^2 + (M_o - 0)^2}$$

and since from Eq. 13, $(L_o - 1)^2 = (a_1/a_2)^2 M_o^2$,

$$r = \sqrt{M_o^2\left[\left(\frac{a_1}{a_2}\right)^2 + 1\right]} = \sqrt{\frac{\left(\frac{a_1}{a_2}\right)^2\left(1 - \frac{\operatorname{Re} y_{11}}{a_1}\right)^2}{\left(\frac{a_1}{a_2}\right)^2 + 1}} = \frac{\left|a_1 - \operatorname{Re} y_{11}\right|}{\sqrt{a_1^2 + a_2^2}}$$

Substituting values from Eq. 5,

$$r = \frac{\left|\operatorname{Re}\left(y_{12}y_{21}\right) - 2\operatorname{Re} y_{11}\operatorname{Re} y_{22}\right|}{\left|y_{12}y_{21}\right|} \tag{17}$$

From the similar triangles of Fig. 4,

$$\frac{P'_{io}}{r} = \frac{CP'_{io}}{1}$$

Therefore

$$\boxed{C = \frac{1}{r} = \frac{\left|y_{12}y_{21}\right|}{\left|2\operatorname{Re} y_{11}\operatorname{Re} y_{22} - \operatorname{Re}\left(y_{12}y_{21}\right)\right|}} \tag{18}$$

GAIN RELATIONSHIPS

We now find maximum device gain, G_{max}. From Fig. 4, $P'_o = P'_{oo}(1 - M'^2)$ and $P'_i = P'_{io}(M'/r + 1)$; then the device gain, G, is

$$G = \frac{P'_o}{P'_i} = \frac{P'_{oo}}{P'_{io}}\frac{\left(1 - M'^2\right)}{\left(1 + \frac{M'}{r}\right)} = G_{oo}\frac{\left(1 - M'^2\right)}{\left(1 + \frac{M'}{r}\right)} = KG_{oo} \tag{19}$$

When $G = G_{max}$, let $M' = \overline{M'}$ and $K = K_G$, i.e., let

$$G_{max} = K_G G_{oo} \tag{20}$$

Then $(dG/dM')_{M'=\overline{M'}} = 0$. This gives

$$\overline{M'} = -r\left(1 \pm \sqrt{1 - \frac{1}{r^2}}\right)$$

or since $\overline{M'} > -r$

$$\overline{M'} = -r\left(1 - \sqrt{1 - \frac{1}{r^2}}\right)$$

Substituting from Eq. 18 gives

$$\overline{M'} = -\frac{1}{C}\left(1 - \sqrt{1 - C^2}\right) \tag{21}$$

From Eq. 19 we have

$$K_G = \frac{1 - \overline{M'}^2}{1 + C\overline{M'}}$$

substituting $\overline{M'}$ from Eq. 21, we get

$$\boxed{K_G = \frac{2}{C^2}\left(1 - \sqrt{1 - C^2}\right)} \tag{22}$$

Note in Eq. 22 that the maximum value of K_G, given $C \leq 1$, occurs when $C = 1$. There $K_G = 2$. The maximum gain G_{max}, therefore, cannot be greater than 3 dB over the gain G_{oo} given when $Y_L = y_{22}^*$. Also, K_G cannot be defined for $C > 1$ since some values of Y_L will then cause oscillation.

From Eq. 9 and 5,

$$G_{oo} = \frac{P'_{oo}}{P'_{io}} = \frac{\dfrac{|y_{21}|^2}{4\mathrm{Re}\, y_{22}}}{\mathrm{Re}\, y_{11} - \dfrac{\mathrm{Re}(y_{12}y_{21})}{2\mathrm{Re}\, y_{22}}}$$

or

$$\boxed{G_{oo} = \frac{|y_{21}|^2}{2\left[2\mathrm{Re}\,y_{11}\mathrm{Re}\,y_{22} - \mathrm{Re}(y_{12}y_{21})\right]}} \quad (23)$$

From Eq. 18 and 19,

$$K = \frac{1 - M'^2}{1 + CM'}$$

or $M'^2 + CKM' = 1 - K$. Therefore

$$\left(M' + \frac{CK}{2}\right)^2 = \left[\sqrt{1 - K + \left(\frac{CK}{2}\right)^2}\right]^2 \quad (24)$$

These are circles centered on the M' axis. Each value of K gives a circle whose center and radius, respectively, are given by

$$\boxed{M'_K = -\frac{C}{2}K}$$

$$\boxed{\rho_K = \sqrt{1 - K + \left(\frac{CK}{2}\right)^2}} \quad (25)$$

Specifically, for $K = K_G$ ($G = G_{max}$).

$$M'_K = -\frac{CK_G}{2} = \overline{M'}$$

$$\rho_{KG} = 0$$

and for $K = 1$ ($G = G_{oo}$).

$$M'_K = -\frac{C}{2}$$

$$\rho_{K=1} = \frac{C}{2}$$

Figures 5 and 6 show constant gain circles constructed from the y parameters of a 2N4957 transistor. A few constant gain circles are shown at 1,000 MHz (Fig. 5) and 450 MHz (Fig. 6). Note particularly that for $C < 1$,

1. A gain of $K_G \cdot G_{oo}$ occurs at only one point (a particular Y_L).

2. A gain of $1 \cdot G_{oo}$ occurs for a circle which includes the point $L = 1$, $M = 0$.

3. Gain circles are contained within one another—the largest circles correspond to the least gain.

and for $C > 1$,

1. As can be seen from Fig. 4, gain corresponding to values of $M' < -r$ are indeterminate. Areas to one side of the $M' = 1/C$ line in Fig. 6 correspond to values of Y_L which will lead to oscillation. Figure 7 shows the gain diagram for such a case.

2. Equation 22 is meaningless; however, Eq. 23, 24, and 25 hold, just as for $C < 1$.

3. All circles intersect at $M' = -1/C$, $L' = \pm\sqrt{1 - 1/C^2}$ (see Appendix B).

We have now found the gain of the device as a function of L and M. Since the L and M values relate to Y_L, we now find gain as a function of Y_L. Equation 8 gave

$$Y_L + y_{22} = \frac{2\text{Re}\, y_{22}}{L + jM}$$

Set

$$Y_L + y_{22} = Y_2 = G_2 + jB_2 \tag{26}$$

Then

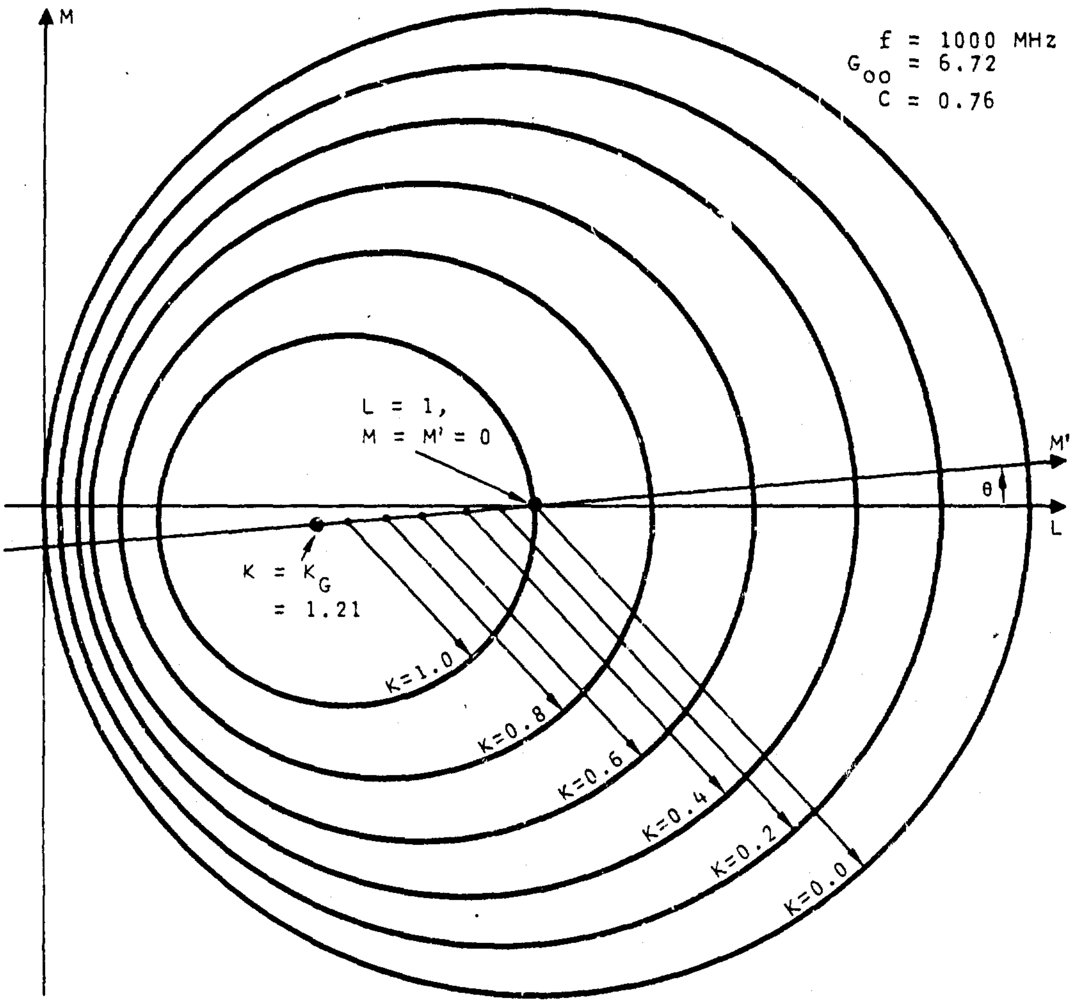

FIG. 5. Constant Gain Circles for C = 0.76.

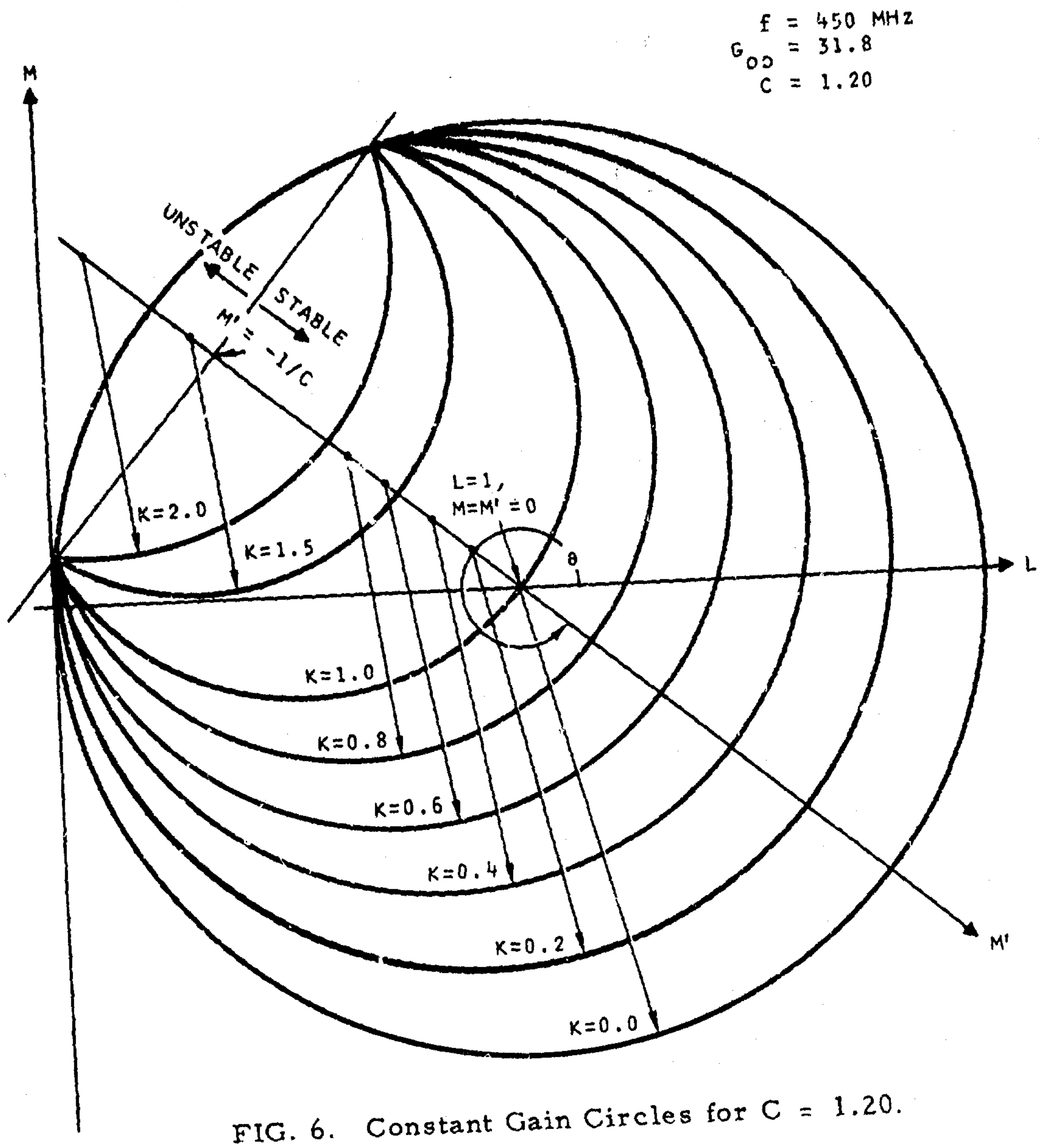

FIG. 6. Constant Gain Circles for C = 1.20.

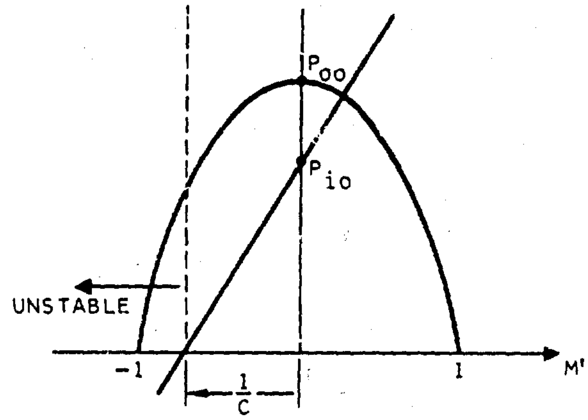

FIG. 7. Gain Diagram for M' < -r.

$$Y_L + y_{22} = \frac{2\text{Re}\, y_{22}(L - jM)}{L^2 + M^2}$$

$$G_2 = \frac{2L\text{Re}\, y_{22}}{L^2 + M^2}$$

$$B_2 = \frac{-2M\text{Re}\, y_{22}}{L^2 + M^2}$$

or

$$L^2 - \frac{2L\text{Re}\, y_{22}}{G_2} + M^2 = 0$$

$$M^2 + \frac{2M\text{Re}\, y_{22}}{B_2} + L^2 = 0$$

which can be written

$$\left(L - \frac{\text{Re}\, y_{22}}{G_2}\right)^2 + M^2 = \left(\frac{\text{Re}\, y_{22}}{G_2}\right)^2$$

$$\left(M + \frac{\mathrm{Re}\, y_{22}}{B_2}\right)^2 + L^2 = \left(\frac{\mathrm{Re}\, y_{22}}{B_2}\right)^2 \tag{27}$$

APPLICATION STEPS

Equations 27 represent orthogonal circles, very similar to those obtained by relating reflection coefficient to load impedances of a transmission line (Smith Chart). The circles obtained from Eq. 27 are plotted in Fig. 8 and 9 over the gain circles of Fig. 6 and 7, respectively. Of course, one need only to use the ready-made circles on a Smith Chart. The design process is therefore as follows:

1. Normalize a Smith Chart in terms of $\mathrm{Re}\, y_{22}$ (or $\mathrm{Re}\, h_{22}$) as shown in Fig. 9. Be sure to add 1.0 to all values of the abscissa of the Smith Chart.

2. Obtain sets of Y parameters for the center of the frequency range to be used. For broadband amplifiers several sets may be required (for each set plot one chart).

3. Draw the gradient line at an angle $\theta = \tan^{-1} \mathrm{Arg}\, (-y_{12}y_{21})^*$.

4. Find $C = |y_{12}y_{21}| / |2\mathrm{Re}\, y_{11}\mathrm{Re}\, y_{22} - \mathrm{Re}\, (y_{12}y_{21})|$.

5. If $C > 1$, draw a line perpendicular to the gradient line at a distance $-1/C$ from the center of the chart.

6. If $C < 1$, find $K_G = 2\left(1 - \sqrt{1 - C^2}\right)/C^2$

7. Find $G_{oo} = |y_{21}|^2 / 2(2\mathrm{Re}\, y_{11}\mathrm{Re}\, y_{22} - \mathrm{Re}\, y_{12}y_{21})$.

8. Plot circles of radii $\rho_K = [1 - K + (KC/2)^2]^{\frac{1}{2}}$ centered at $M'_K = CK/2$.

9. Determine ranges of L and M values needed for given bandwidth. This determines required Y_L and also input admittance (Appendix C).

10. Devise matching networks for source and load impedances.

Steps 9 and 10 are best carried out on a digital computer, especially when a large bandwidth is required. Several sets of G_2, B_2 values may

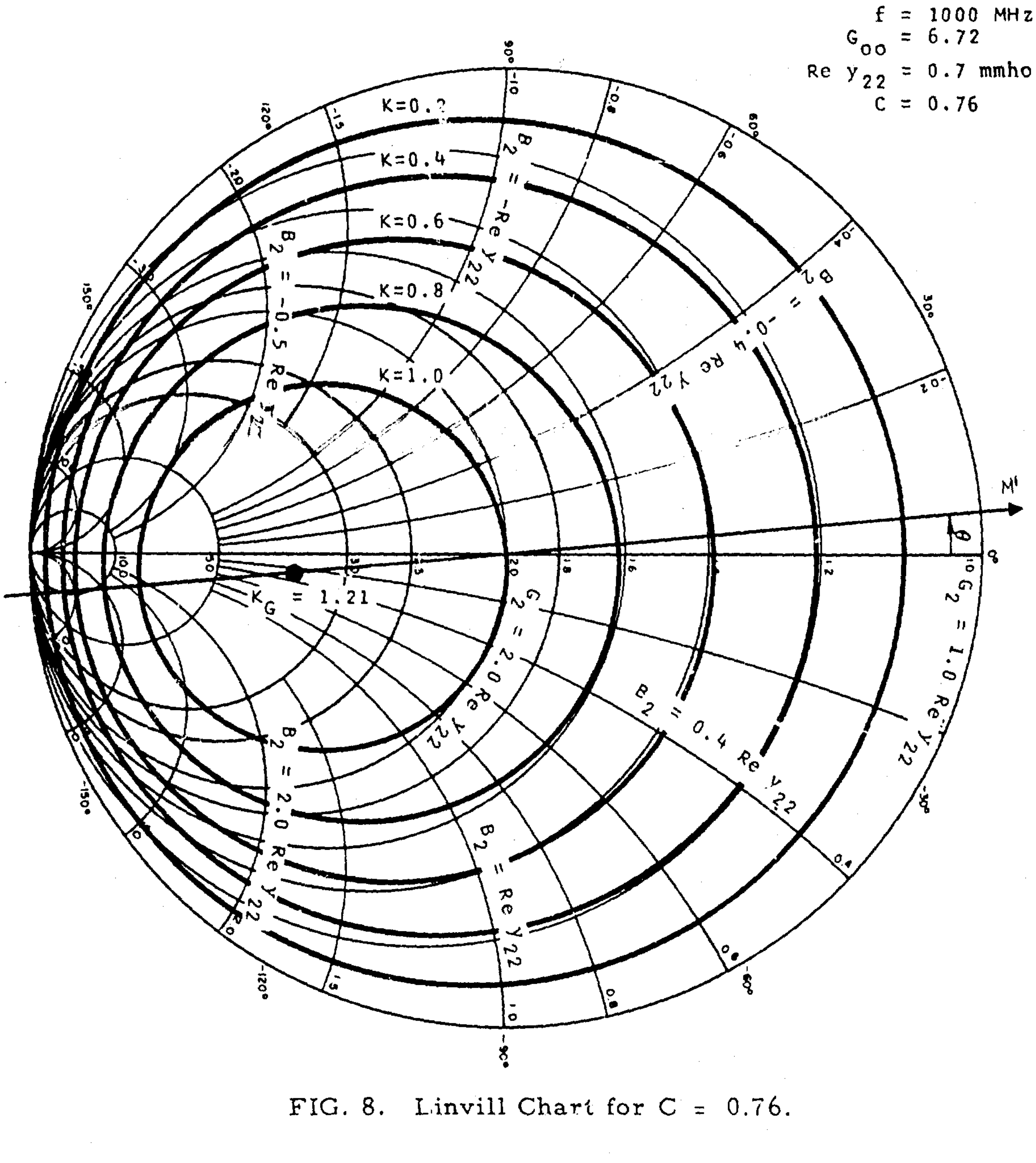

FIG. 8. Linvill Chart for C = 0.76.

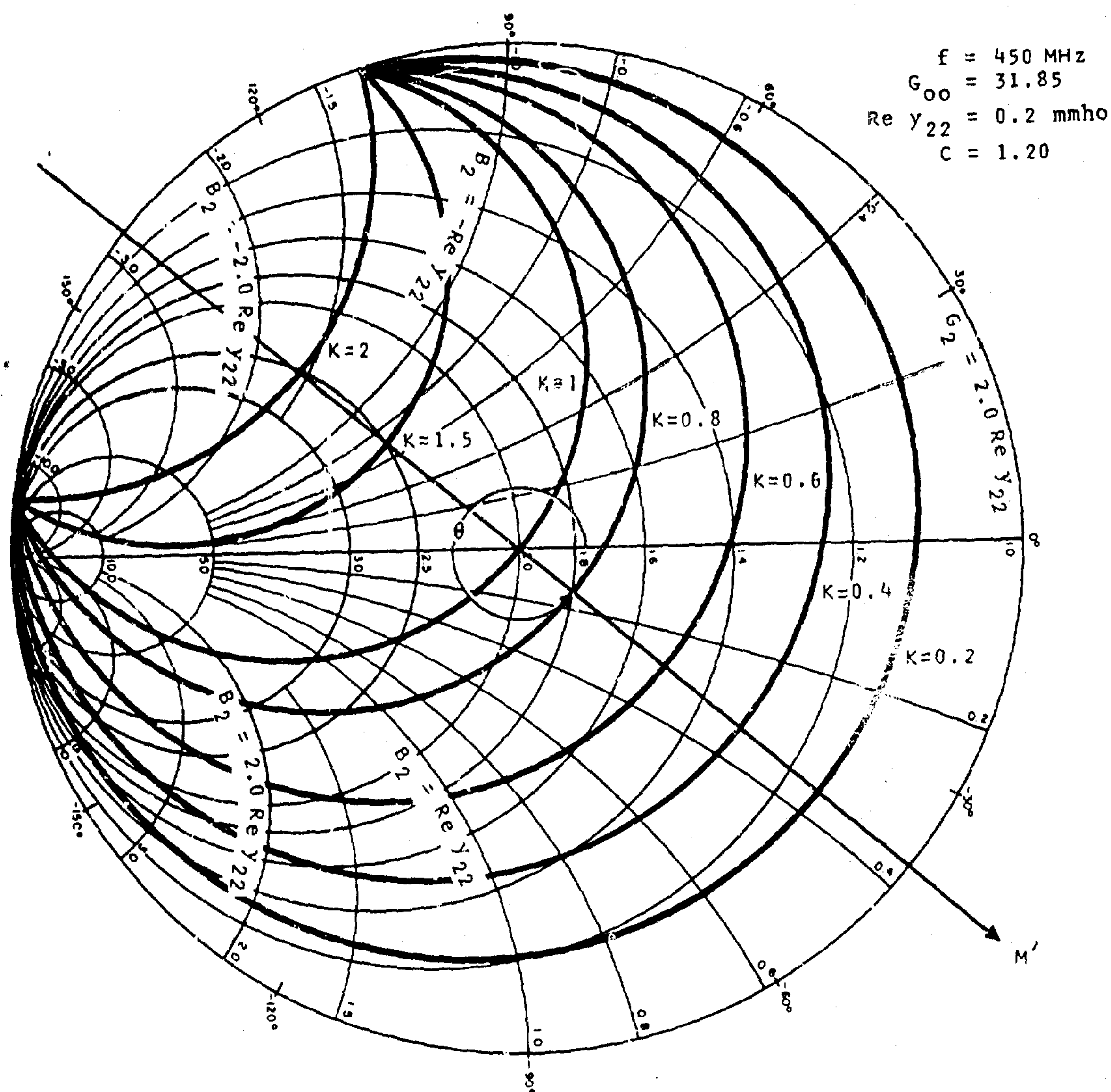

FIG. 9. Linvill Chart for C = 1.20.

be selected, e.g., from within the K = 1 (Gain = G_{oo}) circle of a Linvill chart near the highest frequency of interest. Each G_2, B_2 set requires a particular output admittance. At this frequency a matching network is devised which makes the load appear as the required output admittance. The input admittance of the amplifier is also determined by the G_2, B_2 set under consideration. An input network is then devised that makes this input admittance appear as the conjugate of the source admittance. We now have one frequency at which we know that all available power will be absorbed by the amplifier and a given power gain will appear at the load. At other frequencies the output network and load, together with the new values of y parameters, determine new G_2, B_2 values and hence new device gain. The resulting transistor input admittance and input network will determine how much available power will be absorbed by the amplifier. The network attenuation multiplied by the device gain yields total power gain of the amplifier as a function of frequency. For some of the original G_2, B_2 values selected at the high frequency, the resultant mismatches at lower frequencies will yield a rather level total-gain-versus-frequency curve.

Appendix A

CONVERSION TO h PARAMETERS

In this appendix the procedure is shown for deriving the transistor power equations using the h parameters.

By definition

$$e_1 = h_{11}i_1 + h_{12}e_2$$

$$i_2 = h_{21}i_1 + h_{22}e_2 = -e_2Y_L$$

Therefore,

$$\frac{e_2}{i_1} = -\frac{h_{21}}{h_{22} + Y_L}$$

If Y_L were set $= h_{22}^*$,

$$\left(\frac{e_2}{i_1}\right)_{Y_L=h_{22}^*} = -\frac{h_{21}}{h_{22} + h_{22}^*} = -\frac{h_{21}}{2\mathrm{Re}\,h_{22}}$$

But since Y_L is not necessarity h_{22}^*, set

$$\frac{e_2}{i_1} = (L + jM)\left(-\frac{h_{21}}{2\mathrm{Re}\,h_{22}}\right)$$

where L and M are real. Now

$$P_i = \mathrm{Re}\left(e_1, i_1^*\right)$$

$$= \mathrm{Re}\left\{\left[h_{11}i_1 + h_{12}\left(\frac{-h_{21}}{h_{22} + Y_L}\right)i_1\right]i_1^*\right\}$$

$$= \mathrm{Re}\left\{\left[h_{11}i_1 - \frac{h_{12}(L + jM)h_{21}i_1}{2\mathrm{Re}\,h_{22}}\right]i_1^*\right\}$$

in which e, i are in rms values. We then define

$$P_i' \equiv \frac{P_i}{\left|i_1\right|^2} = \mathrm{Re}\left\{h_{11} - (L + jM)\left[\mathrm{Re}\left(\frac{h_{12}h_{21}}{2\mathrm{Re}\,h_{22}}\right) + j\mathrm{Im}\left(\frac{h_{12}h_{21}}{2\mathrm{Re}\,h_{22}}\right)\right]\right\}$$

Therefore, the P_i' equation in terms of h parameters is

$$P_i' = \mathrm{Re}\,h_{11} - L\,\mathrm{Re}\left(\frac{h_{12}h_{21}}{2\mathrm{Re}\,h_{22}}\right) + M\,\mathrm{Im}\left(\frac{h_{12}h_{21}}{2\mathrm{Re}\,h_{22}}\right) \qquad (28)$$

P_o is given by

$$P_o = -\mathrm{Re}\left(i_2^* e_2\right) = -\mathrm{Re}\Bigg(\Bigg\{h_{21}^* i_1^* + h_{22}^*\left[\frac{-h_{21}^*}{2\mathrm{Re}\,h_{22}}(L - jM)i_1^*\right]\Bigg\}(L + jM)\left(\frac{-h_{21}}{2\mathrm{Re}\,h_{22}}\right)i_1\Bigg)$$

from which

$$P_o' = \frac{P_o}{\left|i_1\right|^2} = -\mathrm{Re}\left[-\frac{\left|h_{21}\right|^2}{2\mathrm{Re}\,h_{22}}(L + jM) + \frac{\left|h_{21}\right|^2}{\left(2\mathrm{Re}\,h_{22}\right)^2}\left(L^2 + M^2\right)h_{22}^*\right]$$

$$= -\mathrm{Re}\left[-\frac{\left|h_{21}\right|^2}{2\mathrm{Re}\,h_{22}}(L + jM) + \frac{\left|h_{21}\right|^2\left(L^2 + M^2\right)}{\left(2\mathrm{Re}\,h_{22}\right)^2}\left(\mathrm{Re}\,h_{22} - j\mathrm{Im}\,h_{22}\right)\right]$$

or

$$P'_o = L\frac{|h_{21}|^2}{2\mathrm{Re}\,h_{22}} - \left(L^2 + M^2\right)\frac{|h_{21}|^2}{2\mathrm{Re}\,h_{22}} \tag{29}$$

Equations 28 and 29, it can be noted, are identical to Eq. 3 and 4 if we set

$$a = \frac{y_{12}y_{21}}{2\mathrm{Re}\,y_{22}} = \frac{h_{12}h_{21}}{2\mathrm{Re}\,h_{22}}$$

$$b_1 = \frac{|y_{21}|^2}{2\mathrm{Re}\,y_{12}} = \frac{|h_{21}|^2}{2\mathrm{Re}\,h_{12}}$$

The analysis follows through with $h_{ij} \Rightarrow y_{ij}$, including the normalization of the Smith Charts to $\mathrm{Re}\,h_{22}$.

Appendix B

CIRCLE INTERSECTION POINTS FOR $C > 1$

In this appendix it is shown that the power gain circles for $C > 1$ all intersect at ($M' = -1/C$, $L' = \pm\sqrt{1 - 1/C^2}$).

Equations 25 gives

$$\rho_K = \sqrt{1 - K + \left(\frac{CK}{2}\right)^2} \qquad M'_K = -\frac{C}{2}K$$

The equations for circles of different K values (see Fig. 10) can be written

$$(L')^2 + \left(M' - M'_K\right)^2 = \rho_K^2$$

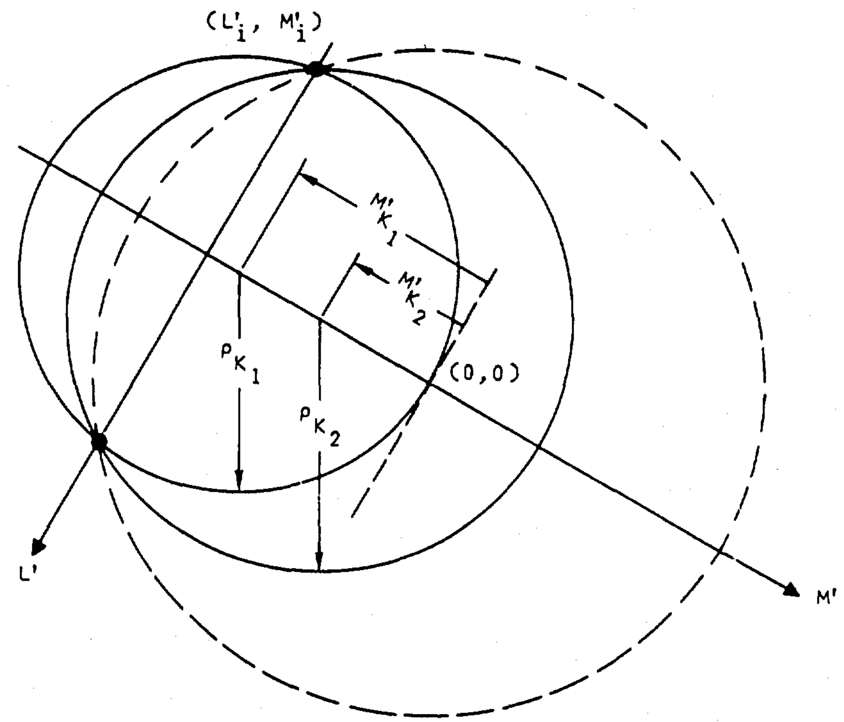

FIG. 10. Gain Circle Intersections.

Substituting for ρ_K and M'_K, we get

$$(L')^2 + (M')^2 + CM'K = 1 - K$$

If the intersection of two circles due to K_1 and K_2 occur at L'_i and M'_i, we will have

$$\left(L'_i\right)^2 + \left(M'_i\right)^2 + CM'_iK_1 = 1 - K_1$$

$$\left(L'_i\right)^2 + \left(M'_i\right)^2 + CM'_iK_2 = 1 - K_2$$

Solving simultaneously, we find that

$$M'_i = -\frac{1}{C}$$

$$L'_i = \pm\sqrt{1 - \frac{1}{C^2}} \qquad (30)$$

Since M'_i and L'_i are independent of K values, all circles for $C > 1$ will intersect at

$$M'_i = -\frac{1}{C} \qquad L'_i = \pm\sqrt{1 - \frac{1}{C^2}}$$

and $(L'_i)^2 + (M'_i)^2 = 1$, i.e., the intersections are those points where the $M' = -1/C$ line intersects the extremities of the Smith Chart.

Appendix C

DETERMINATION OF INPUT ADMITTANCE FROM G_2 AND B_2 VALUES

The following procedure shows how the input admittance Y_{in} can be determined from the coordinates of a point (G_2, B_2) on the Linvill chart.

Since, by Eq. 1,

$$e_1 = y_{11}e_1 + y_{12}e_2$$

$$e_2 = e_1(L + jM)\left(-\frac{y_{21}}{2\mathrm{Re}\, y_{22}}\right)$$

Then

$$Y_{in} = \frac{i_1}{e_1} = y_{11} - (L + jM)\left(\frac{y_{12}y_{21}}{2\mathrm{Re}\, y_{22}}\right) \tag{31}$$

As in Eq. 5, let

$$a = a_1 + ja_2 = \left(\frac{y_{12}y_{21}}{2\mathrm{Re}\, y_{22}}\right) = |a|e^{j\phi'} = |a|\cos\phi' + j|a|\sin\phi'$$

Then $\tan\phi' = a_2/a_1 = \tan\phi = -\tan\theta$ (Eq. 14 and 15). From Fig. 3,

$$\sin\phi = \sin\theta$$

$$\cos\phi = -\cos\theta$$

Therefore,

$$a = -|a|(\cos\theta - j\sin\theta)$$

Substitution in Eq. 31 then gives

$$Y_{in} = y_{11} + (L + jM)|a|(\cos\theta - j\sin\theta)$$

or

$$\frac{Y_{in} - y_{11}}{|a|} = (L + jM)(\cos\theta - j\sin\theta) \tag{32}$$

$$= (L\cos\theta + M\sin\theta) + j(M\cos\theta - L\sin\theta) = M'' + jL''$$

where M", L" are orthogonal axes rotated an angle θ from the L, M axes. The projections M", L" of the point (G_2, B_2) onto these axes thereby give (see Fig. 11)

$$Y_{in} = y_{11} + |a|(M'' + jL'') = G_{in} + jB_{in}$$

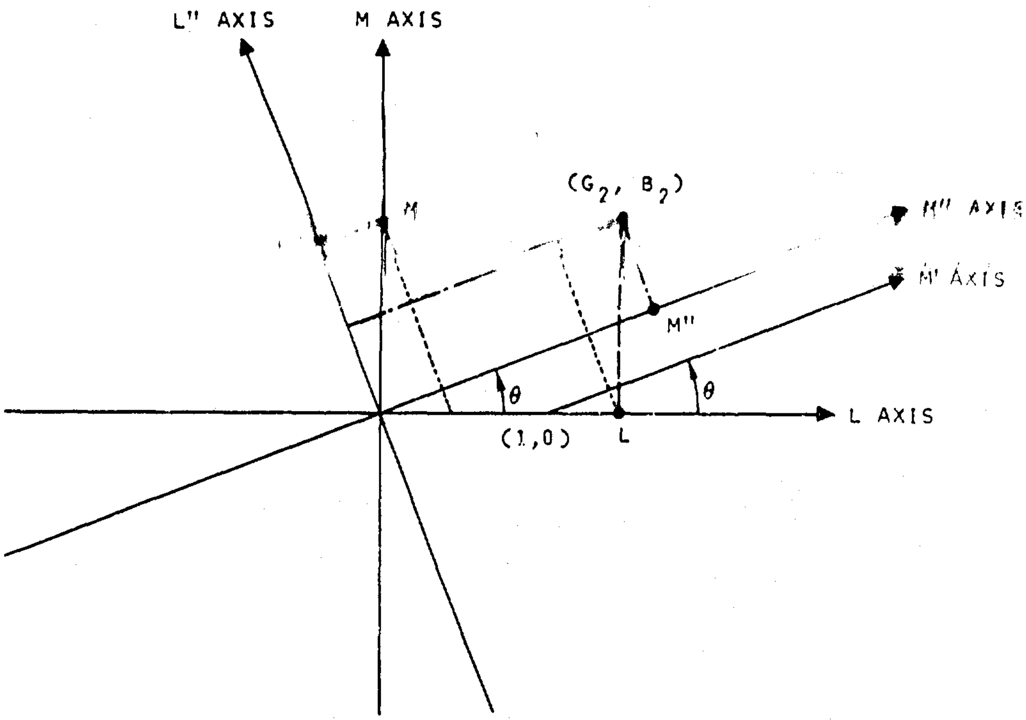

FIG. 11. Point Coordinates for Two Sets of Axes.

where G_{in} and B_{in}, the input conductance and susceptance, respectively, are

$$G_{in} = \mathrm{Re}\, y_{11} + |a| M''$$

$$B_{in} = \mathrm{Im}\, y_{11} + |a| L''$$

and

$$a = \left| \frac{y_{12} y_{21}}{2 \mathrm{Re}\, y_{22}} \right|$$

REFERENCES

1. Linvill, J. G., and L. G. Schimpf. "The Design of Tetrode Transistor Amplifiers," BELL SYST TECH J, Vol. 35 (July 1956), pp. 813-840.

2. Linvill, J. G., and J. F. Gibbons. Transistors and Active Circuits. New York, McGraw-Hill, 1961.

3. Johnson, G. "High Frequency Amplifier Design Using Admittance Parameters," ELECTRO-TECHNOL (November 1963), pp. 74-79.

4. Johnson, G., P. Norris, and F. Opp. "High Frequency Amplifier Design Using Admittance Parameters," ELECTRO-TECHNOL (December 1963), pp. 66-74.

5. Lauchner, J., and M. Silverstein. "Design High Frequency Amplifiers Graphically," ELECTRON DESIGN (April 1966), pp. 48-54.

6. Texas Instruments, Inc. Solid State Communication, by the Engineering Staff. New York, McGraw-Hill, 1966.

Security Classification UNCLASSIFIED

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Weapons Center Corona Laboratories Corona, California 91720	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE
THE LINVILL METHOD OF HIGH FREQUENCY TRANSISTOR AMPLIFIER DESIGN

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

5. AUTHOR(S) (First name, middle initial, last name)
David Rubin

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March 1969	31	6

8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. AIRTASK A36533/216/69F17343604 d.	9a. ORIGINATOR'S REPORT NUMBER(S) NWCCL TP 845 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT
THIS DOCUMENT IS SUBJECT TO SPECIAL EXPORT CONTROLS AND EACH TRANSMITTAL TO FOREIGN GOVERNMENTS OR FOREIGN NATIONALS MAY BE MADE ONLY WITH PRIOR APPROVAL OF THE COMMANDING OFFICER OF THE NAVAL WEAPONS CENTER CORONA LABORATORIES (CODE 2035), CORONA, CALIFORNIA 91720.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Air Systems Command Navy Department Washington, D. C. 20360

13. ABSTRACT

This report discusses a method—known as the Linvill method—for determining the terminations that a transistor amplifier should have for a specified value of power gain and bandwidth. Basically, the Linvill method makes use of measured transistor parameters to develop charts from which one can read power gain and input impedance or admittance as functions of the load termination. The report gives a complete geometrical derivation of the Linvill "stability factor," whose value is an indication of the stability of the amplifier under various load conditions. In addition, procedure steps are given for using the charts developed for determining input and load admittances.

Security Classification UNCLASSIFIED

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Linvill method						
transistor amplifiers						
admittance parameters						
Smith Chart						
stability factor						

SUPPLEMENTARY

INFORMATION

NOTICES OF CHANGES IN CLASSIFICATION, DISTRIBUTION AND AVAILABILITY

69-20 15 October 1969

IDENTIFICATION	FORMER STATEMENT	NEW STATEMENT	AUTHORITY
AD-850 232 Naval Weapons Center, Calif. Corona Labs. Rept. no. NWCCL-TP-845 Mar 69	No Foreign without approval of Commanding Officer, Naval Weapons Center, Corona Labs., Attn: Code 3035, Corona, Calif.	No limitation	USNWC ltr, 16 Apr 69